MEMORANDUM
RM-5870-SSSA
NOVEMBER 1968

AD 683683

NUMERICAL MODELS OF A TROPICAL CUMULUS CLOUD WITH BILATERAL AND AXIAL SYMMETRY

F. W. Murray

PREPARED FOR:
DEPARTMENT OF COMMERCE

The RAND Corporation
SANTA MONICA • CALIFORNIA

38

MEMORANDUM
RM-5870-ESSA
NOVEMBER 1968

NUMERICAL MODELS OF A TROPICAL CUMULUS CLOUD WITH BILATERAL AND AXIAL SYMMETRY

F. W. Murray

This research is supported by the Environmental Science Services Administration, Department of Commerce, under Contract No. E-37-67 (N). Views and conclusions expressed herein should not be interpreted as representing the official opinion or policy of the Department of Commerce.

DISTRIBUTION STATEMENT

This document has been approved for public release and sale; its distribution is unlimited.

The RAND Corporation

Published by The RAND Corporation

PREFACE

As a part of RAND's continuing interest in weather modification in its broadest aspects, the construction and study of numerical models of atmospheric processes on several scales has been emphasized. An important part of this activity has been the development of a model treating the hydrodynamics and thermodynamics of an individual cumulus cell.

A number of investigators have addressed themselves to this problem, and most have been constrained by computer capacity to assume some form of symmetry in order to reduce the problem from three dimensions to two. It has generally been considered that formulation in Cartesian coordinates is satisfactory, although the more complex cylindrical formulation more nearly describes what occurs in nature. Except for a brief discussion by Ogura in 1963, however, the two formulations have not been directly compared. It is the purpose of this study to make such a comparison in greater depth and so to determine the conditions under which each of the formulations is best used.

Other RAND publications related to the present study are RM-5316-NRL, Numerical Simulation of Cumulus Convection; RM-5564-NRL, A Method of Objective Contour Construction; and RM-5582-ESSA, An Annotated Bibliography of Cloud Modeling.

ABSTRACT

Two versions of a numerical model for cumulus convection are compared. One is symmetrical about a vertical plane, the other about a vertical axis. It is found that the axisymmetric model grows more vigorously than the other, but more realistically represents the relations between updraft and downdraft, the shape, and other characteristics. The model, at least with the eddy exchange coefficients used, appears to be deficient in turbulent entrainment. The previous findings of Ogura are generally confirmed and extended.

CONTENTS

PREFACE iii

ABSTRACT v

Section
I. INTRODUCTION 1

II. THE MODEL 4

III. INITIAL CONDITIONS 9

IV. DISCUSSION 10

V. CONCLUSIONS 24

Appendix 27

REFERENCES 29

I. INTRODUCTION

Since the initial numerical experiments in atmospheric thermal convection by Malkus and Witt (1959) and Ogura (1962, 1963) a number of other models of the same general type have been proposed, each emphasizing one or another aspect of the problem. Among these are the models of Ogura and Charney (1962), Lilly (1962), Chou (1962), Chou, Li, Chang, and Kung (1964), Li, Chao, and Hwu (1964), Asai (1964a,b), Murray and Anderson (1965), Nickerson (1965), Takeda (1965), Orville (1965, 1968), Amirov (1966a,b), Inman (1966), Lebedev (1966), and Arnason, Greenfield, and Newburg (1968). These models differ in many details, but nearly all of them have in common the property of two-dimensionality, the full extension to three dimensions being beyond the capacity of computers available during this period.

There have been two principal ways in which the third dimension was eliminated. In some cases Cartesian coordinates were used with the assumption that no property varies in the y-direction, whereas in others cylindrical coordinates were used with the assumption that no property varies in the azimuthal direction. Certain problems (e.g., the squall line) virtually demand the Cartesian approach, but for modeling individual clouds the cylindrical formulation is more appropriate. However, the additional complexity of the cylindrical formulation has led some investigators to use the other, even when it is apparently less suited to the problem at hand. So far as has been ascertained, only Ogura (1963) has studied the same problem with both geometries, and it is on the basis of rather limited comparative data published by him that it has been assumed that the two schemes yield essentially similar results. It is the purpose of the present study to further test this assumption, and to do so it is desirable to make a more detailed comparison of axial and rectilinear symmetry in numerical cloud modeling.

Most of the models mentioned above are based on the Boussinesq approximation or something very close to it, which permits the definition of a stream function and the derivation of a vorticity equation. A number of finite-difference schemes have been devised to solve this

type of equation. The properties of these schemes have been extensively discussed in the literature (see, e.g., Molencamp, 1968) and will not be further discussed here. Almost all of the practical schemes, however, solve the vorticity equation locally in the Eulerian sense, following which the stream function is found by inversion of an elliptic operator, usually by an iterative technique.

Several of the convection models treat a dry atmosphere. This approach can yield some interesting results, but it is much more useful to consider a moist atmosphere in which water can exist in at least the liquid and vapor phases. Murray and Hollinden (1966) have proposed a model that treats water in all three phases. It has been found that in atmospheric thermal convection, release of latent heat has more effect on buoyancy than any other single process. Some models assume pseudoadiabatic processes, i.e., all condensed water is immediately dropped out. Hence a cloud parcel ascends moist-adiabatically, but descends dry-adiabatically without evaporation. Other models, including the one to be described herein, assume reversible processes; i.e., all condensed water is carried with the air and is available later for evaporation. Still better is the assumption that only part of the condensate falls out as rain, but determination of what part this is to be is still a vexing problem. Precise treatment of rain would necessitate detailed modeling of microphysical processes, but to date no program has been developed that can handle both this and cloud-scale dynamics simultaneously; in all probability this must await the next generation of large, parallel-processing computers. Several existing models, however, have parameterized precipitation mechanisms. The most successful parameterization of this type so far is the one developed by Kessler (1967). The model described herein is based on reversible processes without fallout of rain.

The method developed by Ogura (1963) to handle moist air (and later adopted by several other investigators) is as follows: From the basic equations of hydrodynamics and thermodynamics, one derives a prediction equation for the meridional component of vorticity and conservation equations for total water substance and specific entropy of moist air. The finite-difference analogs of these equations are solved at each

grid point, and the temperature is then determined from the specific entropy by one of two methods, depending on whether or not the air is saturated. Total water substance is then partitioned between liquid and vapor on the assumption that any excess of mixing ratio over the saturation value represents liquid water.

In the present investigation a somewhat different method was employed. The vorticity equation and its solution are essentially the same as in Ogura's model, but the thermodynamic equations are solved in a Lagrangian manner. The rationale for this is that the physical processes relating to condensation and evaporation are usually described in terms of changes occurring in individual parcels. The various processes are interrelated in a highly implicit way, but if attention is confined to an individual parcel, they can be treated serially while maintaining the error within acceptable bounds. The Lagrangian approach is conceptually more straightforward than the Eulerian in this case, and in practice it has been found to be more stable computationally. One drawback is that the smoothing resulting from the necessary interpolation acts like an implicit eddy diffusion. This effect, however, appears to be small enough not to be troublesome. Quantities such as total water substance are not exactly conserved, but the loss is quite small.

II. THE MODEL

The Boussinesq equation of motion for moist air (valid if convection is confined to a layer a few kilometers in depth) is

$$\frac{\partial \underset{\sim}{V}}{\partial t} = -\underset{\sim}{V}\cdot\nabla\underset{\sim}{V} - \bar{\alpha}\nabla p' + g\left(\frac{T^{*\prime}}{\overline{T^*}} - r_\ell\right)\underset{\sim}{k} + \nu_M\nabla^2\underset{\sim}{V} \qquad (1)$$

The symbols are defined in the Appendix. The virtual temperature is defined by

$$T^* = \frac{1 + r_v/\epsilon}{1 + r_v}\, T \qquad (2)$$

The Boussinesq equation of continuity is

$$\nabla\cdot\underset{\sim}{V} = 0 \qquad (3)$$

If one takes the curl of (1) and uses (3), there results

$$\frac{\partial \underset{\sim}{\omega}}{\partial t} = \nabla \times (\underset{\sim}{V} \times \underset{\sim}{\omega}) + \nabla \times B\underset{\sim}{k} - \nu_M \nabla \times (\nabla \times \underset{\sim}{\omega}) \qquad (4)$$

where

$$\underset{\sim}{\omega} = \nabla \times \underset{\sim}{V} \qquad (5)$$

and

$$B = g\left(\frac{T^{*\prime}}{\overline{T^*}} - r_\ell\right) \qquad (6)$$

Two different two-dimensional models derived from (4) will be discussed. The first uses a Cartesian coordinate system (x,y,z) and proceeds

on the assumption that the component of velocity and the variation of all properties in the y-direction are zero; this is the rectilinear model. The second uses a cylindrical coordinate system (x,θ,z)* and proceeds on the assumption that the tangential component of velocity is zero and all properties are independent of θ; this is the axisymmetric model. Subscripts R and A will be used to denote variables pertaining to the rectilinear and axisymmetric models, respectively.

Because of the incompressibility implied by (3) it is possible to express the wind components in terms of a stream function, as follows:

$$
\left.
\begin{aligned}
u &= -\frac{\partial \psi_R}{\partial z} = -\frac{1}{x}\frac{\partial \psi_A}{\partial z} \\
w &= \frac{\partial \psi_R}{\partial x} = \frac{1}{x}\frac{\partial \psi_A}{\partial x}
\end{aligned}
\right. \tag{7}
$$

The horizontal component of vorticity is

$$
\eta = -\left[\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\right] \tag{8}
$$

or

$$
\left.
\begin{aligned}
\eta_R &= -\left[\frac{\partial^2 \psi_R}{\partial x^2} + \frac{\partial^2 \psi_R}{\partial z^2}\right] \\
\eta_A &= -\left[\frac{\partial}{\partial x}\left(\frac{1}{x}\frac{\partial \psi_A}{\partial x}\right) + \frac{1}{x}\frac{\partial^2 \psi_A}{\partial z^2}\right]
\end{aligned}
\right\} \tag{8a}
$$

---

*The coordinate x is used here rather than the more conventional r to facilitate comparison with the Cartesian system.

The transverse or tangential component of (4) is then

$$\left.\begin{aligned}\frac{\partial \eta_R}{\partial t} &= -\left[\frac{\partial}{\partial x}(u\eta_R) + \frac{\partial}{\partial z}(w\eta_R)\right] - \frac{\partial B}{\partial x} + \nu_M\left[\frac{\partial^2 \eta_R}{\partial x^2} + \frac{\partial^2 \eta_R}{\partial z^2}\right] \\ \frac{\partial \eta_A}{\partial t} &= -\left[\frac{\partial}{\partial x}(u\eta_A) + \frac{\partial}{\partial z}(w\eta_A)\right] - \frac{\partial B}{\partial x} + \nu_M\left[\frac{\partial}{\partial x}\left\{\frac{1}{x}\frac{\partial}{\partial x}(x\eta_A)\right\} + \frac{\partial^2 \eta_A}{\partial z^2}\right]\end{aligned}\right\} \quad (9)$$

If at a given time ψ (which determines u, w, and η) and T, r_v, and r_ℓ (which collectively determine B) are known at each grid point, (9) can be solved to determine η at each grid point at a later time, following which (8a) can be solved for ψ at the later time. The latter can be done as a Dirichlet problem with $\psi = 0$ on all boundaries, a condition equivalent to the requirement that no air pass through any of the boundaries. (The boundaries are taken to be two horizontals, one representing the ground and one an undisturbed level, and two verticals, one representing an axis of symmetry and one a lateral wall enclosing the region of convective circulation.)

The Lagrangian part of the scheme encompasses the determination of T, r_v, and r_ℓ at each grid point at the later time. This is done in several steps, as follows:

1. Using the values of u and w at a grid point and its neighbors at the beginning and end of the time step, a single-time-step backward trajectory is computed to determine the point of origin of the air parcel that is found at the grid point at the end of the time step.

2. Values of temperature, humidity, etc. at the point of origin at the beginning of the time step are found by interpolation among the four surrounding grid points.

3. If the air parcel at the point of origin is subsaturated in the presence of liquid water or is supersaturated, an adjustment is made to T, r_v, and r_ℓ to rectify this condition. Since the adjustment is small, it is assumed to occur instantaneously at constant pressure. The saturation mixing ratio is given b

$$r_s = \frac{\epsilon e_s}{p - e_s} \tag{10}$$

and the saturation vapor pressure by Tetens' formula:

$$e_s = 6.1078 \exp\left[\frac{a(T - 273.16)}{(T - b)}\right] \tag{11}$$

with a = 17.2693882 and b = 35.86. Differentiation and combination of (10) and (11) yield

$$dr_s = \frac{r_s(\epsilon + r_s)a(273.16 - b)}{\epsilon(T - b)^2} dT \tag{12}$$

But the change of temperature due to condensation or evaporation is

$$dT = \frac{L}{c_p + r_v c_{pv} + r_\ell c_w} dr_\ell = - \frac{L}{c_p + r_v c_{pv} + r_\ell c_w} dr_v \tag{13}$$

The object of the adjustment is to make r_s and r_v equal; hence

$$r_s + dr_s = r_v + dr_v \tag{14}$$

Combination of (12), (13), and (14) yields

$$dr_\ell = -dr_v = \frac{r_v - r_s}{1 + \dfrac{r_s(\epsilon + r_s)a(273.16 - b)L}{\epsilon(T - b)^2(c_p + r_v c_{pv} + r_\ell c_w)}} \tag{15}$$

The static adjustment consists of applying (15) to r_ℓ and r_v and (13) to T. If $r_v < r_s$, the adjustment can be no greater than r_ℓ, (15) notwithstanding.

4. If the parcel is now exactly saturated, a dynamic adjustment is made to T, r_v, and r_ℓ. Under the assumption of conservation of

equivalent potential temperature, it can be shown that the rate of change of saturation mixing ratio is

$$\frac{dr_s}{dt} = \frac{1 - \frac{c_p T - Lr_s}{\epsilon L}}{L + \frac{c_p RT^2}{Lr_s(\epsilon + r_s)}} gw \qquad (16)$$

If $r_v = r_s$, then $dr_s/dt = -dr_v/dt = dr_\ell/dt$, and so (16) gives the rate of condensation. Thus one can determine the change of T, r_v, and r_ℓ due to condensation or evaporation as the parcel moves from its point of origin to the grid point.

5. The temperature of the parcel also changes adiabatically at the rate $-gw/c_p$; this must be taken into account whether or not step (4) was applicable. At this stage the values of T, r_v, and r_ℓ resulting from the application of the various adjustments to the values determined for the point of origin at the beginning of the time step are preliminary values for the grid point at the end of the time step.

6. Fickian eddy diffusion is assumed, so terms of the form $\nu_T \nabla^2 T'$, $\nu_r \nabla^2 r_v$, and $\nu_r \nabla^2 r_\ell$ must be included in the rates of changes of temperature, mixing ratio of vapor, and mixing ratio of liquid. These terms are evaluated in the Eulerian manner at each grid point at the beginning of the time step, and are now used to adjust the grid-point values of T', r_v, and r_ℓ at the end of the time step.

7. Because of nonlinearities in most of the processes, there may at this stage be points at which the air is subsaturated in the presence of liquid water or is supersaturated. If so the adjustment of step (3) is again applied, this time at the grid point.

III. INITIAL CONDITIONS

It is assumed that initially the atmosphere is at rest and horizontally homogeneous. Ordinarily the variation of T and r_v in the vertical is in accordance with some actual atmospheric sounding, and r_ℓ is taken to be zero everywhere. Unless some impulse is applied, the right-hand side of (9) is identically zero, and nothing can happen. The usual impulse used for this type of model is a bubble of increased temperature. This makes B no longer independent of x, so η can grow. The rising motion so induced in the center of the bubble can lead to condensation and conversion of latent heat to sensible heat, making the process self-sustaining. Something like this process is thought to occur in nature when a cumulus cloud is generated, but in one respect this process is unrealistic. The arbitrary initial increase in temperature within the bubble is accompanied by a decrease in relative humidity, delaying the onset of condensation and ensuring that the cloud develops on the driest part of the region.

It is perhaps more realistic to introduce a humidity impulse rather than a temperature impulse. There is then a horizontal gradient of virtual temperature (though not of temperature) to make the right-hand side of (9) nonzero. If the impulse is such as to produce saturation within the bubble, condensation can start immediately. Humidity impulses have been studied both theoretically and observationally by Vul'fson (1963), who found them to be of great importance, especially in convection over a water surface. This is the type of impulse used in the present study.

IV. DISCUSSION

Ogura (1963) presented results of several runs with a model having axial symmetry. He used a 30-by-30 grid, and for lateral boundary conditions he required that both $u = 0$ and $\partial w/\partial x = 0$ at the central axis and at the outer wall. One run having a basic atmosphere with a conditionally unstable lapse rate and saturation throughout was repeated with rectilinear geometry, but with a 20-by-20 grid and cyclic lateral boundary conditions.

The main points noted by Ogura in comparing the two runs were:

a. The strength of the downdraft outside the cloud is stronger in the rectilinear model than in the axisymmetric. At 8 minutes the ratios of maximum downdraft to maximum updraft were .25 and .06, respectively.

b. The heating of the subsiding air produced a temperature gradient such that a cell with reversed circulation appeared below the cloud in the rectilinear model, but not in the axisymmetric.

c. The greater excess temperature produced in the downdraft of the rectilinear model resulted in appearance of a region of positive vorticity. This did not occur with the axisymmetric model.

It is possible that Ogura's use of different boundary conditions and grid dimensions contributed to the different results noted in the models with the two geometries. In order to avoid this possibility in the present investigation, runs were made with the two geometries using identical initial data, impulses, boundary conditions, and grid dimensions. The initial sounding used was one from San Juan, Puerto Rico, for a day on which well-developed cumuli were observed in the region. A humidity impulse was chosen so that initially there was slight supersaturation near the central axis and the convective condensation level; hence condensation commenced immediately, and the model cloud grew vigorously.

The most obvious difference in the response of the axisymmetric and rectilinear models is that the former developed much faster and more strongly. This is apparent from Fig. 1, which shows the magnitude of the maximum updraft as a function of time. By 8 minutes the

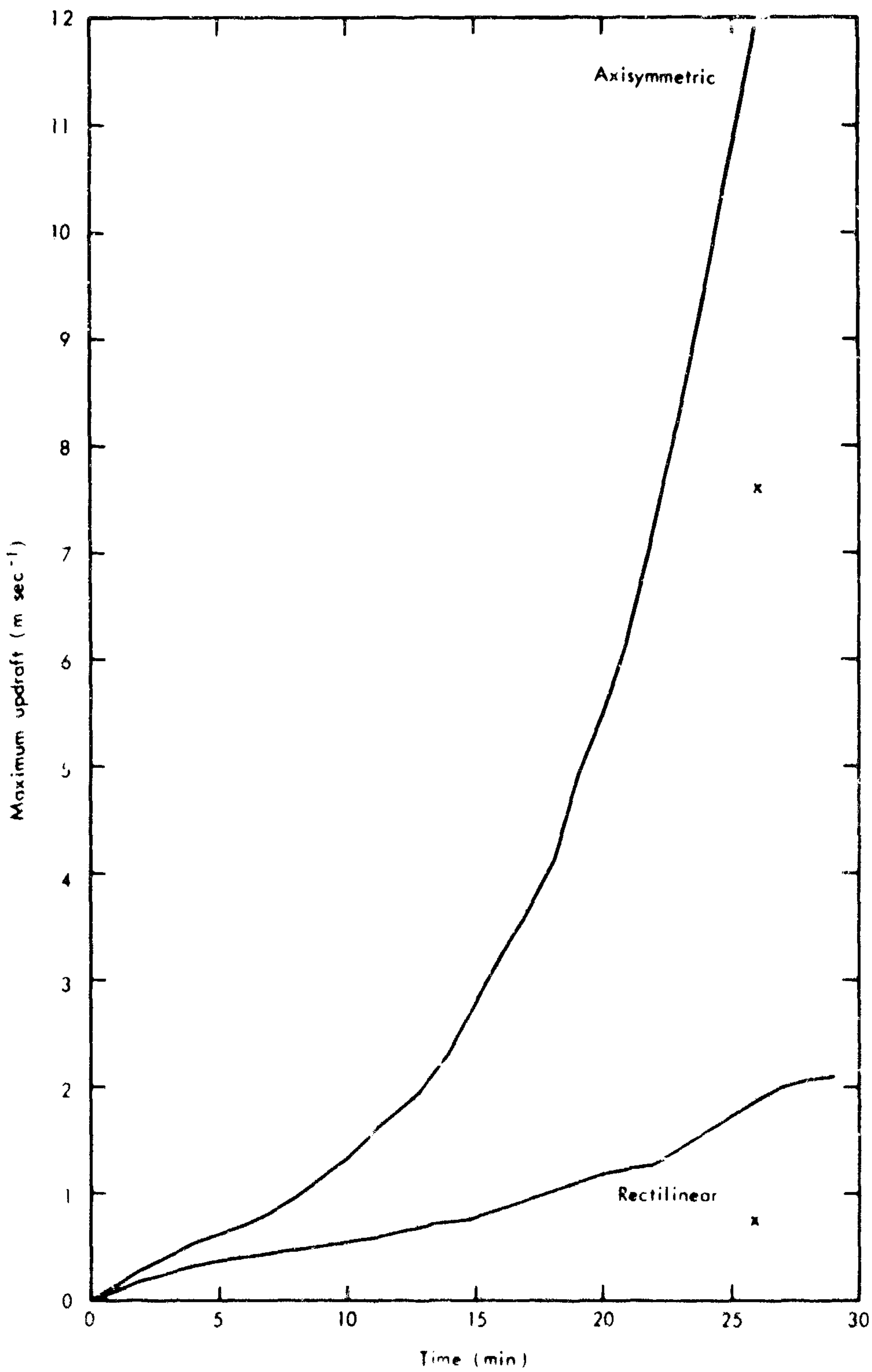

Fig. 1 -- Magnitude of maximum updraft

axisymmetric updraft was twice as strong as the rectilinear, and it continued to increase exponentially until computation was stopped at 26 minutes, when the updraft was nearly 12 m sec^{-1}. By contrast, the updraft of the rectilinear model did not quite reach 2 m sec^{-1} in the same time, and its rate of growth showed signs of leveling off at 29 minutes.

This behavior could be predicted qualitatively from the nature of the two geometries. Under the assumption that the horizontal area within radial distance X is πX^2 for the axisymmetric model and 2XY for the rectilinear model, where Y is some distance along the y-axis, one can determine that if there is updraft from the axis out to X_1 and thereafter downdraft out to X_2, the proportion of the total horizontal area occupied by updraft is X_1^2/X_2^2 for the axisymmetric model and X_1/X_2 for the rectilinear. In the present experiment, X_2 = 5800 m for both cases, whereas X_1 is typically 800 m for the axisymmetric case and 600 m for the rectilinear. Thus the updraft covers 2 percent of the total area in the axisymmetric case and 10 percent in the rectilinear. Obviously, if the two updrafts are to transport similar amounts of air through a given level, that of the axisymmetric model must be considerably stronger than the other. It should be remarked, however, that in both models most of the downdraft is concentrated near the evaporating edge of the cloud, and the remainder of the downdraft area has a very slight vertical motion, and even some pockets of weak updraft.

In his initial exposition of the slice method, Bjerknes (1938) showed that the ratio of updraft to downdraft area must be small, and in an elaboration of that work, Petterssen (1939) suggested that 10 percent might be a reasonable figure, although he did not express high confidence in that estimate. However, it agrees with the present results for the rectilinear model. Recently Krishnamurti (1968) determined by a study of total moisture convergence over an area of synoptic scale in the Caribbean and by airborne radar observations that 1 percent of the total area was covered by active convective cloud elements. This agrees with results of the axisymmetric model in the present experiment and offers a degree of confirmation of the superiority of that model in this respect.

One difficulty that occurs throughout this problem is the mathematical singularity on the central axis in the axisymmetric model. This has caused the greatest trouble in the evaluation of w from (7) where $x = 0$. The best method that has been found is to assume, with Ogura, that $\partial w/\partial x = 0$ at $x = 0$ (a reasonable assumption for axial symmetry) whereby it is easily shown that $w = \partial^2\psi_A/\partial x^2$ at $x = 0$. With this formulation the magnitude of w seems to increase sharply, by comparison with the rectilinear model, over the last grid interval as the axis is approached, but not so sharply as with other axisymmetric formulations. The maximum updrafts shown in Fig. 1 are all on the central axis, but the nature of Fig. 1 would not be changed if the magnitude of the maximum updraft not on the central axis had been plotted instead. The effect at 26 minutes would be to reduce the value from 11.9 m sec^{-1} to 7.6 m sec^{-1} in the axisymmetric model and from 1.88 m sec^{-1} to 0.77 m sec^{-1} in the rectilinear model. These points are indicated on the figure by the symbol X.

Ogura did not discuss the actual magnitude of the vertical currents but only the ratio of maximum downdraft to maximum updraft. These ratios for the present experiment are shown in Fig. 2. As Ogura reported, the ratio for the rectilinear model is substantially larger than that for the axisymmetric model, but the ratios are not at all constant. Both show a marked decrease for the first 7 or 8 minutes, followed by a marked increase. If isotachs of the vertical component of velocity or streamlines are plotted for each time step, it is found that the center in which the maximum downdraft is located during the first 7 or 8 minutes is not the same as that for later times. Initially there is a single-cell circulation pattern centered near $x = 1600$ m, with updraft for smaller values of x and downdraft for larger values. As the simulated cloud develops, this cell strengthens and its center moves inward and upward, but a weak secondary center develops and moves outward. At first the maximum downdraft is outside the secondary center, but gradually the downdraft between the centers increases, and after about 8 minutes it becomes the dominant downdraft. Thereafter the maximum updraft and maximum downdraft are closely related to each other and to the area of condensation, although secondary maxima and minima occur at various

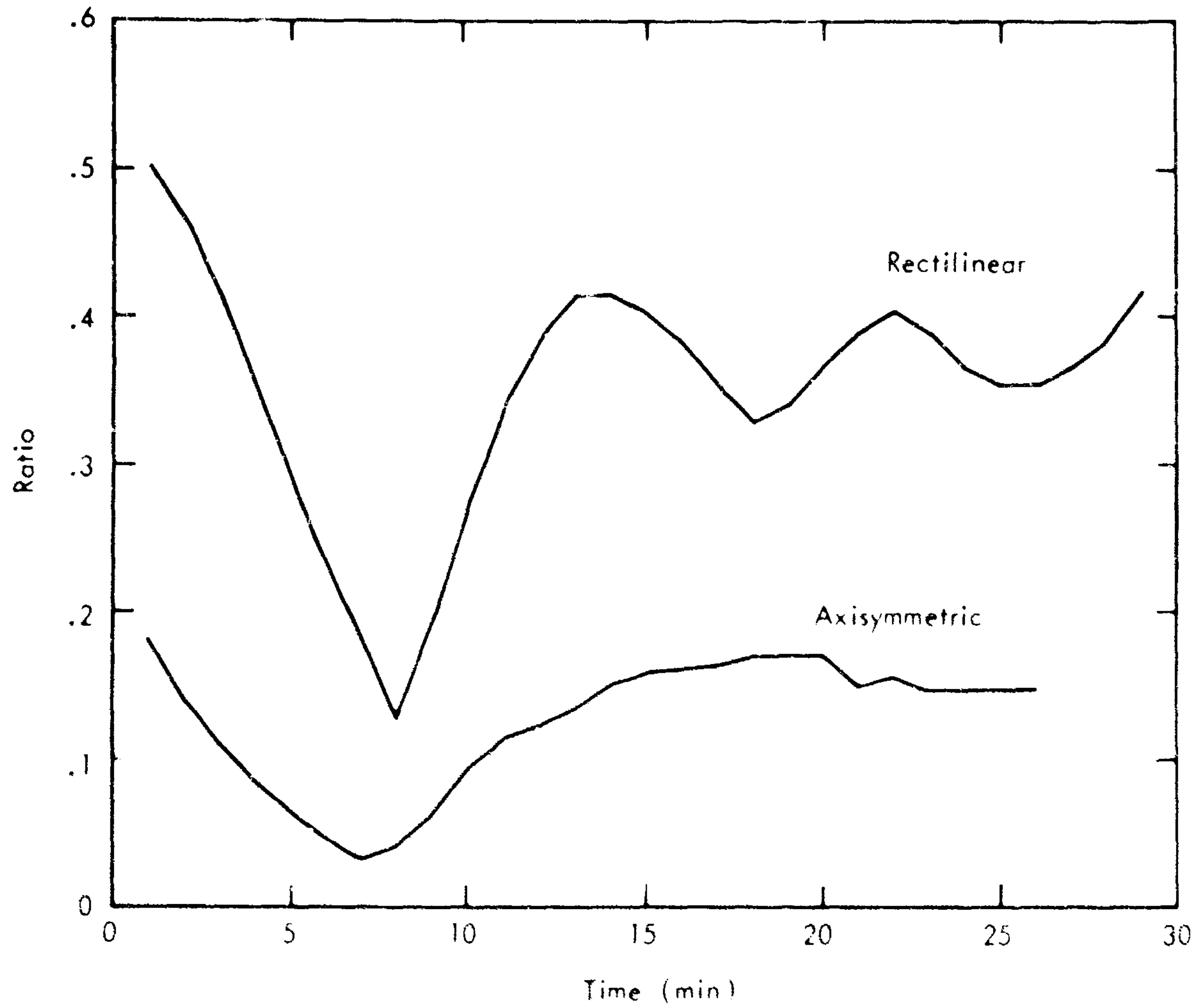

Fig. 2 -- Ratio of magnitude of maximum downdraft to magnitude of maximum updraft

spots throughout the computational region. Hence after about 10 minutes comparisons should be significant.

The ratios shown in Fig. 2 are generally comparable to the ratios of .25 and .06 reported by Ogura at 8 minutes, but they tend to be somewhat higher. The ratio of the ratios in Ogura's experiment was 4:1, whereas in the present experiment it is 3:1 or 2:1. These differences, which do not appear to be significant, can be ascribed to differences in the models, or more particularly to differences in initial data. What is important is that the ratio of maximum downdraft to maximum updraft is larger in the rectilinear model than it is thought to be for actual clouds, whereas it has a more realistic value in the axisymmetric model.

Study of the streamlines and isotachs has disclosed no apparent reason for the regular oscillation of the upper curve of Fig. 2 after 8 minutes, other than that it is connected in some way with the gravity waves of period near 10 minutes that are typical of this convective model. The waves are probably the consequence of the assumption of a rigid upper boundary. It is not known why the lower curve of Fig. 2 fails to show the same oscillation.

Plots of temperature departure as a function of position at various times (e.g., Fig. 3*) show a strong maximum in the region of condensation, surmounted by a cap of temperature deficit resulting from forced lifting of yet-unsaturated air above the active cell. This cap tends to extend downward in a narrow band along the edge of the cloud, where evaporation is taking place. Farther out there is another maximum, strong in the rectilinear model and weak in the axisymmetric, related to subsidence of relatively dry air. It is this temperature excess together with the temperature deficit associated with dry adiabatic ascent below the cloud that cause the development of the reverse-circulation cell mentioned by Ogura. In the present experiment this cell

---

*Figures 3, 4, 5, and 7 are taken from computer-constructed charts. On them small squares indicate grid points of maximum value relative to adjacent surrounding grid points, and small circles represent relative minima in the same sense. The ordinate is height and the abscissa is radial distance, both in meters. The edge of each square block is five mesh lengths.

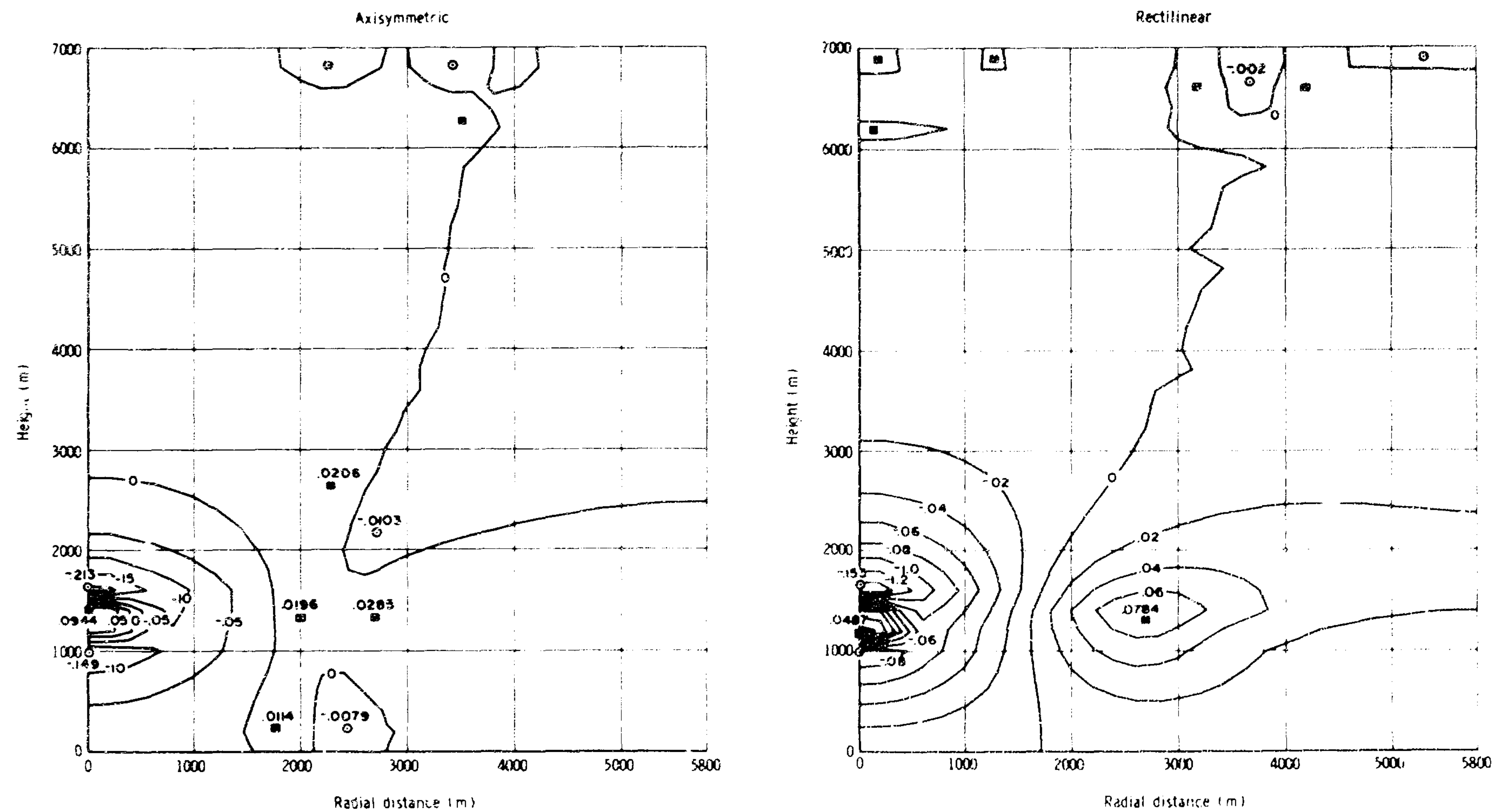

Fig. 3 -- Temperature departure at 5 minutes, case of weak development

appeared both with the axisymmetric and the rectilinear models, but it is very weak in both (see Fig. 4). It anything, it is a little stronger in the axisymmetric model, but so are all other circulations. A strong reverse cell has appeared in previous runs of the rectilinear model, however, when the initial impulse was a temperature excess as in Ogura's experiment. It is believed that its failure to appear strongly here is related to the type of impulse.

The center of excess temperature in the subsiding air has another effect that has been evident in most models of this type. As the model cloud develops, its active vortex moves upward, leaving a relatively passive mass of cloud at lower levels. The inflow associated with the vortex brings in warm, dry air, causing that portion of the cloud to evaporate. There frequently results a rather globular cloud in the region of the active vortex, having a narrow stem extending down along the central axis. Because the rectilinear model has a stronger subsidence temperature excess, this effect is particularly noticeable in models with such a geometry; see Ogura's Fig. 20. The same action occurs in the axisymmetric model, but it is less intense. The effect is illustrated in Fig. 5, which is taken from a case with a more extensive initial impulse than that of Figs. 3 and 4, and consequently with more vigorous development. In many instances the stem of the cloud in the rectilinear model is only a single mesh unit wide.

The continued maintenance of a narrow stem below the active part of the cloud brings one to a consideration of entrainment. In other types of cloud models an explicit entrainment term is included, which generally has the effect of mixing some of the air from a uniform environment with uniform cloud air, resulting in a cloud with new uniform properties. In the type of model considered here the cloud is not an entity, but only a collection of grid points having in common the single property $r_{\ell} > 0$. Hence, the ordinary type of entrainment term is not pertinent.

There are, however, two types of entrainment in this model. Dynamic entrainment refers to the process whereby a particle trajectory leads from a point outside the cloud to a point within it. This occurs when a parcel without liquid water reaches, through some process, a

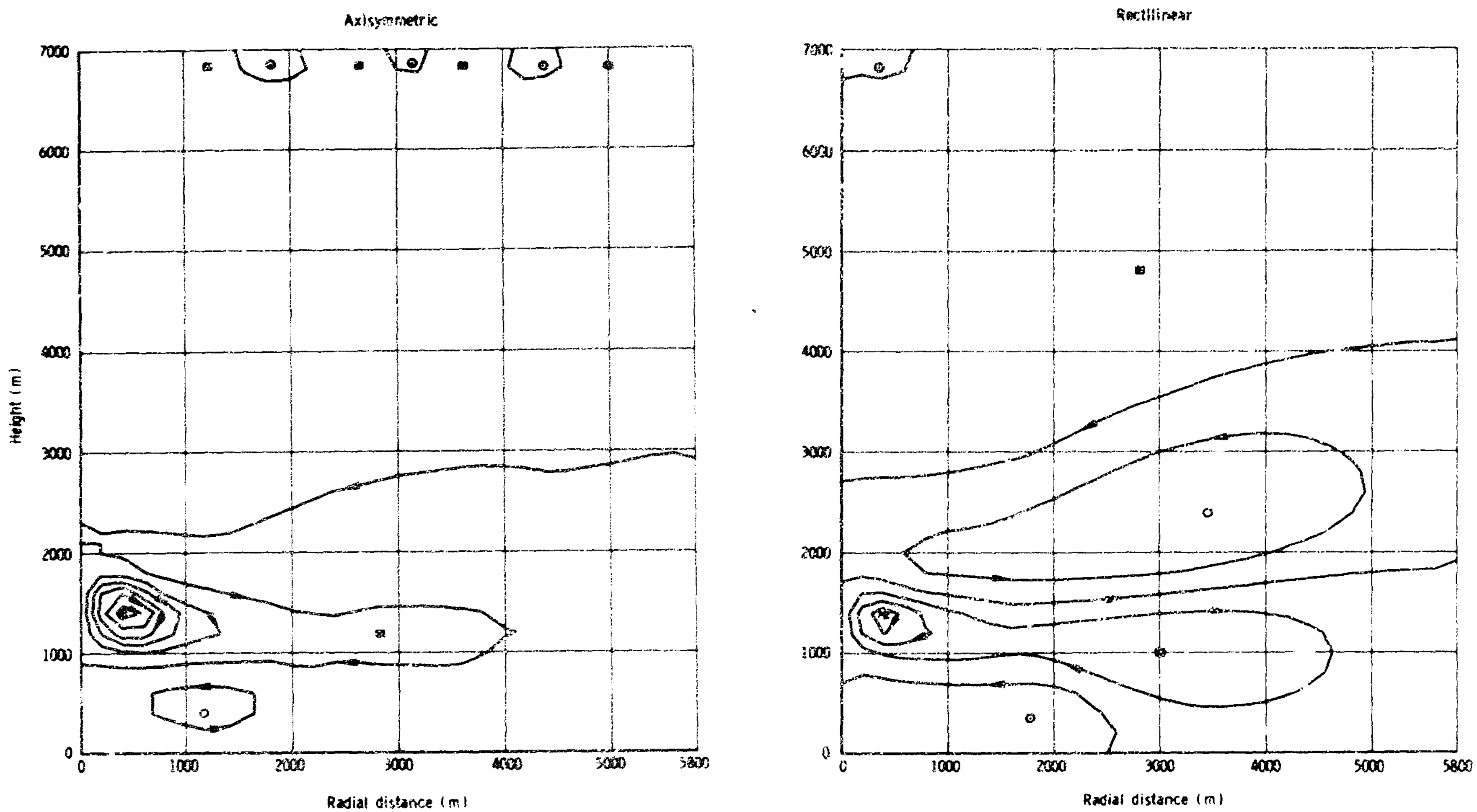

Fig. 4 -- Streamlines at 12 minutes, case of weak developme t

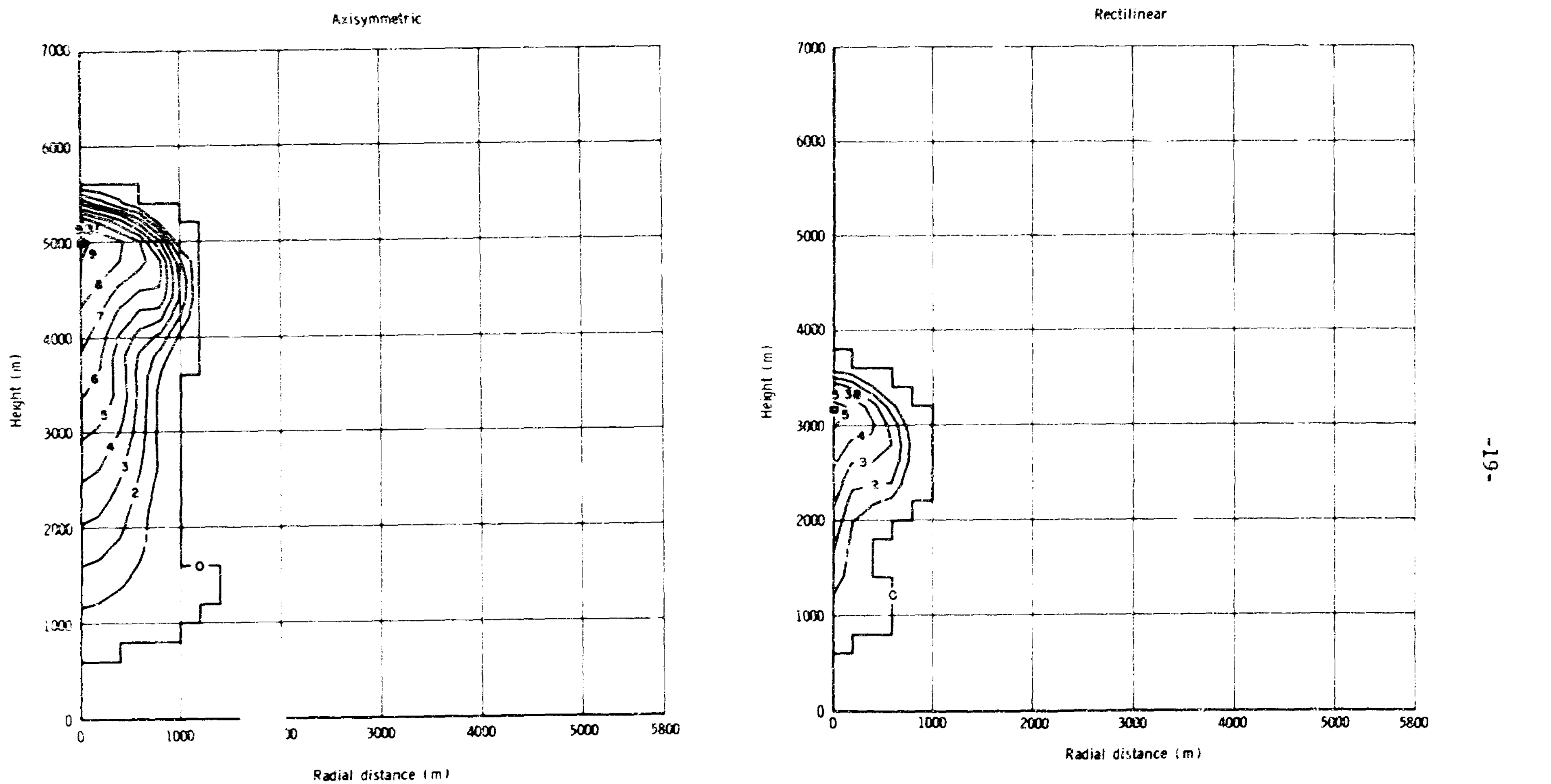

Fig. 5 -- Mixing ratio of liquid, 16 minutes, case of strong development

state of saturation, and condensation commences. Such a parcel has then been entrained. Turbulent entrainment is more comparable to the normal notion of entrainment--a form of mixing. In this model it can occur explicitly through the Fickian eddy diffusion terms or implicitly through the interpolation required by the Lagrangian part of the computation.

Since $u = 0$ at $x = 0$, dynamic entrainment cannot occur along the central axis (except trivially at the bottom and top of the cloud). It can, however, occur at all other points about the cloud, and it creates the mushroom effect noted above. Turbulent mixing, on the other hand, can occur at any point of the cloud, including the central axis, and it may be the key to control of unrestrained growth both in the model and in nature. The basic sounding used in the present experiment, the lower part of which is shown in Fig. 6, is only slightly less stable and more humid than the mean August Caribbean sounding. A few cumulonimbus clouds were observed over the ocean at the time of this sounding, but most of the clouds grew only to moderate heights (Simpson, Simpson, Andrews, and Eaton, 1965). Yet the parcel method, which does not allow for entrainment, would predict that any disturbance would grow to the tropopause. It is apparent that nature uses entrainment to check runaway convection.

If in the present model the eddy exchange coefficients are zero, the development along the central axis is almost identical to that of the parcel method. Computed soundings at the later stages of development show an almost dry-adiabatic lapse rate below the cloud and a saturated-adiabatic lapse rate within the cloud. This, of course, does not hold away from the central axis, where dynamic entrainment occurs. The present computer runs were made with $\nu_M = \nu_T = \nu_r = 40 \text{ m}^2 \text{ sec}^{-1}$, the smallest value Ogura found to have any discernible effect on the computations.* It is planned to repeat the computations with larger

*Molencamp (1968) showed that with Ogura's finite-difference scheme there was also an implicit eddy exchange equivalent to the explicit exchange with $\nu_M = 40$. The present finite-difference scheme is different, but the effect is probably comparable.

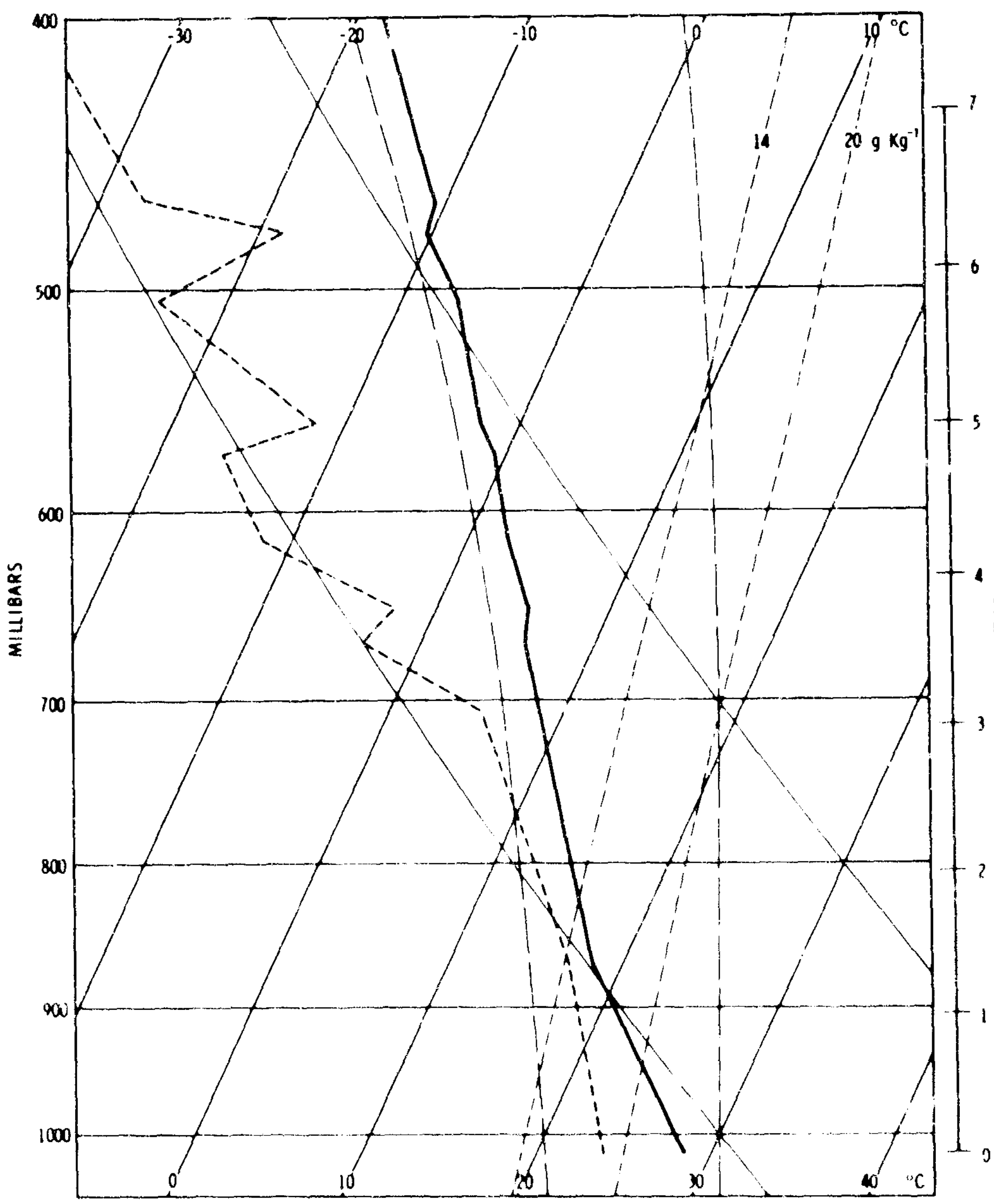

Fig. 6 -- Sounding for San Juan, Puerto Rico, 2300Z, 20 August 1963

and perhaps variable coefficients to see whether this will curb the model's tendency toward excessive updrafts and temperature departures within the cloud. This is particularly important for the axisymmetric model, for its strong vertical wind along the axis can lead to computational instability and the necessity to terminate computation before the cloud has reached its mature stage.

Some types of models, such as that of Simpson, Simpson, Andrews, and Eaton (1965), require specification of cloud diameter, which controls the rate of entrainment, and consequently determines the height to which the cloud will rise. In the present model, by contrast, cloud diameter is one of the results that come out of the computation. It may be related to the diameter of the initial impulse, but in the many runs that have been made it has invariably been less than that diameter. It is interesting to note that with identical initial impulses the diameter of the axisymmetric cloud turns out to be a little greater than that of the rectilinear cloud. The formulation of this model, however, does not ensure that there will be lesser entrainment for a greater diameter, except in that the wider simulated cloud has many more interior grid points, but only a few more boundary grid points. Thus the ratio of mass entrained to total mass is less, but this is not a quantity that is considered by the model.

Ogura's observation of the development of a center of positive vorticity in connection with the temperature maximum associated with subsidence seems not to be verified in the present experiment. Instead, the vorticity in both models appears to be organized into several horizontal layers of like sign (in the lower part of the computational region at least), the vorticity center associated with the active cloud circulation being embedded in one of them. This is illustrated in Fig. 7, which, incidentally, has its sign convention opposite from Ogura's. The vorticity chart for the axisymmetric model shows a stronger principal maximum and more noise than that for the other model, but they are essentially similar.

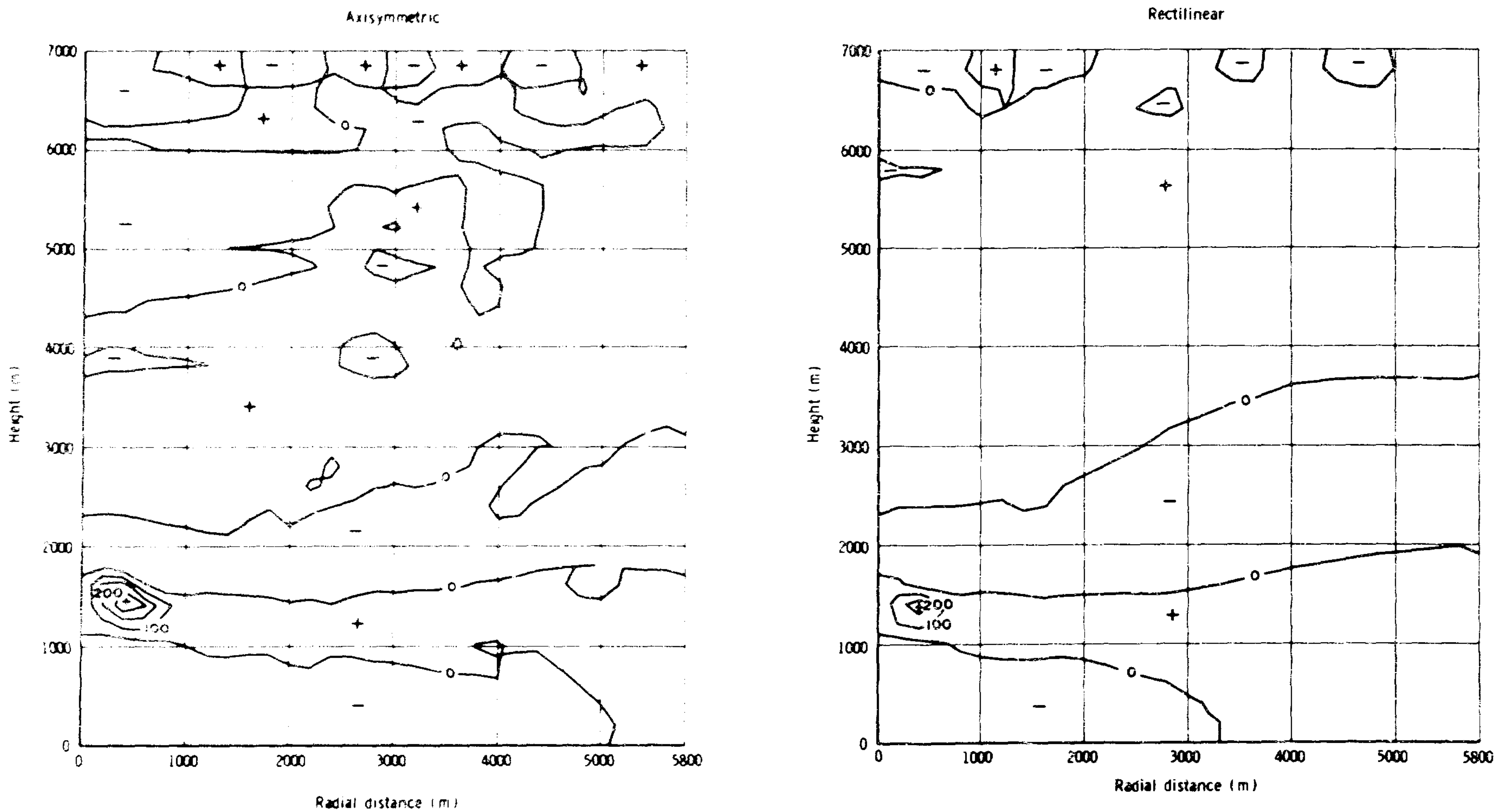

Fig. 7 -- Vorticity at 12 minutes, case of weak development

V. CONCLUSIONS

Despite certain fundamental differences between the two approaches, most notably in the computation of temperature and moisture content, Ogura's model and the present one give essentially similar results. In particular, Ogura's observations on the differences between models with axisymmetric and rectilinear symmetry are generally borne out by the present experiment, even though Ogura's comparative runs differed from each other in more than the basic geometry. Ogura, however, presented only fragmentary results of the comparison, whereas the present study involves a somewhat more systematic investigation into the effects of basic geometry on the model.

The principal conclusion reached by Ogura is that the downdraft is relatively stronger in the rectilinear model than in the axisymmetric. This is true also of the present study, for the ratio of maximum downdraft to maximum updraft is about .35 to .40 for the rectilinear model and near .15 for the other. These ratios are not constant with time, however, and their physical significance seems to change about the eighth minute. At later times, however, the comparisons are consistent.

What is even more striking than the difference in ratio of downdraft to updraft is the difference in speed and strength of development. In the present experiment the axisymmetric model showed much stronger updrafts than the rectilinear, and consequently its model cloud was both bigger and wetter. As a matter of fact, its growth was such that the updrafts became unrealistically large, requiring termination of computation before the model cloud could reach its mature stage. This was partly the result of the instability and high humidity of the initial sounding used. Experience has shown that a well-placed stable layer can easily overcome the unrestrained growth. It has also shown that the rate of growth, though not its final extent, can be controlled by varying the size of the initial impulse. These observations apply mainly to the axisymmetric model; the much slower rectilinear model was not run long enough in this experiment to develop unrealistic updrafts, although the indications were that it would eventually do just that. Ogura did not report this particular difficulty, but his computational region was so small that the model cloud impinged on the upper

boundary, making further computation unrewarding from the standpoint of realism. The implication is that the present model did not contain sufficient turbulent entrainment to slow down its development. It appears that through turbulent entrainment, nature is able to prevent runaway growth of convective clouds in moist, unstable air. This approach will be investigated in the course of future work with the model, but the strong difference between the two geometries remains, and is likely to continue to do so.

Aside from the lack of dynamic entrainment on the central axis, a sort of feedback mechanism seems to ensure the faster growth of the updraft of the axisymmetric model. As was mentioned, the built-in convergence of the coordinate system ensures that the updraft will initially be relatively stronger than the downdraft. When condensation occurs, there is a considerable increase of temperature, and the contribution to (9) of the buoyancy term becomes large in the limited region of the cloud. This leads to a stronger updraft, more condensation, and so on. Thus the difference due merely to geometry is magnified. Between the cloud edge and the subsidence temperature maximum, however, the effect of the buoyancy term is negative, but it is not enhanced by condensation or evaporation. Moreover, the convergence of the coordinate systems is less, and the two models are more nearly alike. The result is that even though the rectilinear model shows a more pronounced subsidence temperature maximum, the pattern of the vorticity distribution is not much different in the two models, nor is the pattern of the streamlines, though there are differences in the spacing of the streamlines. This result, however, may be more closely connected with the initial impulse than with the geometry. Ogura noted a distinct difference in the vorticity patterns and the development of a reverse-flow cell beneath the cloud in the rectilinear model. The present model, when run with a temperature impulse similar to that of Ogura rather than one of humidity, does indeed show the development of such a reverse cell more strongly with the axisymmetric geometry than with the rectilinear. On the whole, though, it appears that the humidity impulse leads to more realistic results, so it has been retained.

Taken together, the results of the present experiment confirm Ogura's results, but go beyond them to suggest that the two geometries lead to strong differences in the simulated clouds. In most respects the axisymmetric model gave results more in keeping with observations of real clouds, but it still needs greater restraint, perhaps through turbulent entrainment, to prevent runaway growth. Further study is suggested in that respect and with regard to the effects of different initial impulses and of some sort of precipitation mechanism.

Appendix

Let φ represent any of the primary variables (e.g., pressure). Then by definition

$$\varphi = \overline{\varphi} + \varphi_B + \varphi'$$

where

$\overline{\varphi}$ is the (constant) mean value of φ over the entire computational region at initial time in the absence of motion.

$\varphi_B = \varphi_B(z)$ is the departure of φ from $\overline{\varphi}$ at a given height z at initial time in the absence of motion.

$\varphi' = \varphi'(x,z,t)$ is the departure of φ from $\overline{\varphi} + \varphi_B$ at any location (x,z) and any time t.

The symbols used in this report have the following meanings:

<u>Scalars</u>

B = buoyancy parameter

L = latent heat of condensation

R = gas constant for dry air

T = temperature

T^* = virtual temperature

c_p = specific heat of dry air at constant pressure

c_{pv} = specific heat of water vapor at constant pressure

c_w = specific heat of liquid water

e_s = saturation vapor pressure

g = acceleration due to gravity

p = pressure

r_ℓ = mixing ratio of liquid water to dry air

r_s = saturation mixing ratio

r_v = mixing ratio of water vapor to dry air

t = time

u = horizontal component of wind

w = vertical component of wind

x radial coordinate

z = vertical coordinate

α = specific volume

ϵ = ratio of molecular weights of water vapor and dry air

η = horizontal component of vorticity

ν_M = coefficient of eddy transport of momentum

ν_r = coefficient of eddy diffusion of water substance

ν_T = coefficient of eddy diffusion of temperature

ψ = stream function

Vectors

$\underset{\sim}{V}$ = wind velocity

$\underset{\sim}{k}$ = vertical unit vector

$\underset{\sim}{\omega}$ = vorticity

REFERENCES

Amirov, A. D., 1966a: "Development of thermals and cumulus clouds in a stratified atmosphere," English translation of Izvestiya Akademii Nauk SSSR, Fizika Atmosfery i Okeana, Vol. 2, published by the American Geophysical Union, pp. 271-279.

Amirov, A. D., 1966b: "On the role of turbulence in convective motion," English translation of Izvestiya Akademii Nauk SSSR, Fizika Atmosfery i Okeana, Vol. 2, published by the American Geophysical Union, pp. 427-431.

Arnason, G., R. S. Greenfield, and E. A. Newburg, 1968: "A numerical experiment in dry and moist convection including the rain stage," Journal of the Atmospheric Sciences, Vol. 25, pp. 404-415.

Asai, Tomio, 1964a: "Numerical experiment of cumulus convection under the pseudo-adiabatic process," Papers in Meteorology and Geophysics, Meteorological Research Institute of Japan, Vol. 15, No. 1, pp. 1-30.

Asai, Tomio, 1964b: "Cumulus convection in the atmosphere with vertical wind shear: numerical experiment," Journal of the Meteorological Society of Japan, Series II, Vol. 42, pp. 245-259.

Bjerknes, J., 1938: "Saturated-adiabatic ascent of air through dry-adiabatically descending environment," Quarterly Journal of the Royal Meteorological Society, Vol. 64, pp. 325-330.

Chou Hsiao-P'ing, 1962: "Development of cumulus clouds," English translation of Izvestiya Akademii Nauk SSSR, Seriya Geofyzicheskaya, No. 4, published by the American Geophysical Union, pp. 358-363.

Chou Hsiao-P'ing, Li Hsing-Sheng, Chang Yao-K'o, and Kung Tseng-Chen, 1964: English translation of Acta Meteorologica Sinica, Vol. 34, pp. 475-484, published in Translations on Communist China: Science and Technology, No. 223, U.S. Department of Commerce, Joint Publications Research Service 32,737, 5 November 1965, pp. 33-51.

Inman, R. L., 1966: The Evolution of Convective Motions in a Rotating Fluid: A Numerical Calculation, Ph.D. Dissertation, Texas A&M University, 90 pp.

Kessler, Edwin, III, 1967: "Model circulations with microphysical processes," On the Continuity of Water Substance, Tech. Memo IERTM-NSSL 33, ESSA National Severe Storms Laboratory, pp. 1-85.

Krishnamurti, T. N., 1968: "A calculation of percentage area covered by convective clouds from moisture convergence," Journal of Applied Meteorology, Vol. 7, pp. 184-195.

Lebedev, S. L., 1966: "A three-dimensional nonstationary model of an atmospheric convection cell with a cloud," English translation of Izvestiya Akademii Nauk SSSR, Fizika Atmosfery i Okeana, Vol. 2, published by the American Geophysical Union, pp. 8-14.

Li Shing-Shong, Chao Jih-P'ing, and Hwu Yün-Chyan, 1964: "A dynamical analysis of the development of the cumulonimbus incus," Scientia Sinica, Vol. 13, pp. 1885-1887.

Lilly, D. K., 1962: "On the numerical simulation of buoyant convection," Tellus, Vol. 14, pp. 148-172.

Malkus, J. S., and G. Witt, 1959: "The evolution of a convective element: a numerical calculation," The Atmosphere and the Sea in Motion, Rockefeller Institute Press and Oxford University Press, New York, pp. 425-439.

Molencamp, C. R., 1968: "Accuracy of finite-difference methods applied to the advection equation," Journal of Applied Meteorology, Vol. 7, pp. 160-167.

Murray, F. W., and C. E. Anderson, 1965: Numerical Simulation of the Evolution of Cumulus Towers, Report SM-49230, Douglas Aircraft Company, Inc., Santa Monica, California, 97 pp.

Murray, F. W., and A. B. Hollinden, 1966: The Evolution of Cumulus Clouds: A Numerical Simulation and its Comparison Against Observations, Report SM-49372, Douglas Aircraft Company, Inc., Santa Monica, California, 147 pp.

Nickerson, E. C., 1965: "A numerical experiment in buoyant convection involving the use of a heat source," Journal of the Atmospheric Sciences, Vol. 22, pp. 412-418.

Ogura, Y., 1962: "Convection of isolated masses of a buoyant fluid: a numerical calculation," Journal of the Atmospheric Sciences, Vol. 19, pp. 492-502.

Ogura, Y., 1963: "The evolution of a moist convective element in a shallow, conditionally unstable atmosphere: a numerical calculation," Journal of the Atmospheric Sciences, Vol. 20, pp. 407-424.

Ogura, Y., and J. Charney, 1962: "A numerical model of thermal convection in the atmosphere," Proceedings of the International Symposium on Numerical Weather Prediction in Tokyo, Meteorological Society of Japan, pp. 431-451.

Orville, H. D., 1965: "A numerical study of the initiation of cumulus clouds over mountainous terrain," Journal of the Atmospheric Sciences, Vol. 22, pp. 684-699.

Orville, H. D., 1968: "Ambient wind effects on the initiation and development of cumulus clouds over mountains," Journal of the Atmospheric Sciences, Vol. 25, pp. 385-403.

Petterssen, S., 1939: "Contribution to the theory of convection," Geofysiske Publikasjoner, Vol. 12, No. 9, 23 pp.

Simpson, J., R. H. Simpson, D. A. Andrews, and M. A. Eaton, 1965: "Experimental cumulus dynamics," Reviews of Geophysics, Vol. 3, pp. 387-431.

Takeda, T., 1965: "The downdraft in convective shower-cloud under the vertical wind shear and its significance for the maintenance of convective system," Journal of the Meteorological Society of Japan, Series II, Vol. 43, pp. 302-309.

Vul'fson, N. I., 1963: "Effect of air humidity on development of convection in a cloudless atmosphere," English translation of Doklady Akademii Nauk SSSR, Vol. 151, (Earth Sciences Sections), published by the American Geological Institute, pp. 10-12.

DOCUMENT CONTROL DATA

1. ORIGINATING ACTIVITY THE RAND CORPORATION	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP

3. REPORT TITLE

NUMERICAL MODELS OF A TROPICAL CUMULUS CLOUD WITH BILATERAL AND AXIAL SYMMETRY

4. AUTHOR(S) (Last name, first name, initial)

Murray, F. W.

5. REPORT DATE	6a. TOTAL No. OF PAGES	6b. No. OF REFS.
November 1968	38	---

7. CONTRACT OR GRANT No.	8. ORIGINATOR'S REPORT No.
E-37-67(N)	RM-5870-ESSA

9a. AVAILABILITY/LIMITATION NOTICES	9b. SPONSORING AGENCY
DDC-1	Department of Commerce

10. ABSTRACT

Comparison of two versions of a numerical model for cumulus convection. One version is symmetrical about a vertical plane; the other, about a vertical axis. It is found that the axisymmetric model grows more vigorously than the other in both speed and strength of development, but more realistically represents the relations between downdraft and updraft. The model, at least with the eddy exchange coefficients used, was found to be deficient in necessary turbulent entrainment. The 1963 findings of Ogura's experiment are generally confirmed but also extended. Further study is suggested on the effects of different initial impulses on cloud growth.

11. KEY WORDS

Models
Weather
Clouds
Meteorology
Computer simulation
Geophysics